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College of Education for Pure Sciences
Department of Mathematics

وزارة التعليم العالي والبحث العلمي
جامعة البصرة
كلية التربية للعلوم الصرفة
قسم الرياضيات

البحوث المنشورة في قسم الرياضيات\ كلية التربية للعلوم الصرفة\ جامعة البصرة للعام الدراسي للعام 2020-2021

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التقويم الذاتي لقسم الرياضيات
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البحوث المنشورة في قسم الرياضيات\ كلية التربية للعلوم الصرفة\ جامعة البصرة للعام الدراسي 2020-2021

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4	The Analysis of Bifurcation Solutions for Camassa-Holm Equation	د. مظهر عبد الواحد عبد الحسين	كلية التربية للعلوم الصرفة	Journal of Informatics and Mathematical Sciences	2019	Scopus
5	Error Estmate for two-dimensional coupled burgers, equations by weak Galerkin finite element method	د. هاشم عبد الخالق كشكول	كلية التربية للعلوم الصرفة	Journal of Physics: Conference Series	2020	Scopus
6	Weak Galerkin finite element method for Solving one-dimensional coupled burgers, equations	د. هاشم عبد الخالق كشكول	كلية التربية للعلوم الصرفة	Springer	2020	Scopus

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Lasso Estimator for High-Dimensional Repeated Measurements Model

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Abstract: In this paper, we propose the lasso method for choice of penalty level and investigate the error of the lasso estimator in repeated measurements model. We introduce our repeated measurements model with its expectation error and its derivative. we investigate the choice of level the penalty parameter for the lasso estimator which plays important role in investigation the lasso estimator and also show that with high probability, the random variable is very close to its expectation. Lastly, we investigate the error of lasso estimator and conclude that with high probability the lasso estimator will pick out most of the important variables.

INTRODUCTION

Many scientists and researchers have been given a definition for the repeated measurements in the different periods of time. [1] defined them as term used to describe the data in which observations of response variable are measured repeatedly for each experimental unit under different experimental conditions. While [2] explained that the repeated measurements require two or more independent groups between the most of known experimental designs in the set of different researches type. In the other words, in the repeated measurements , the observations of experimental units are measured repeatedly in the time unit.

The linear regression model is said to be high-dimensional model when the number of explanatory variables exceeds the number of observations. In other hand, the model is called low-dimensional model when the number of explanatory variables is less than the number of observations. Whatever the linear regression model whether low or high-dimensional, we desire to satisfy some important properties which are: estimation, prediction and variables selection.

A least squares estimator can be obtained by minimizing the residual sum of squares. As long as an inverse of the explanatory matrix exists, this leads to have a unique solution to the given problem. But this method cannot apply on the problem which has high-dimensional data. In the sense that, when the number of unknown coefficients which are to be estimated is larger than the number of sample size. In this case, the uniqueness of solution cannot be obtained. Furthermore the traditional methods, like all possible regression, forward regression, backward regression and step-wise regression cannot be used for the variables selection in the high-dimensional data.

As mentioned above, it is cleared that these methods cannot be applied due to increment in the number of coefficients on the account of sample size. In this case, another methods must be utilized instead of conventional method for the estimation and variables selection. It is known that, the common and suitable method which can be employed to treat with the high-dimensional data is called penalized least squares method. The main use of this method is to overcome the computational problems and also improves the prediction accuracy. The penalized method is based on the principle of reduction the residual sum of squares with the some constraints on the unknown parameters. The estimations of penalized method can be obtained by minimizing an objective function which consists of two parts: loss function and penalty function. The general form of the penalized method is given in the following equation,

$$P_{LS}(\lambda,\beta) = (Y - X\beta)'(Y - X\beta) + P_i(\lambda,\beta),$$

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Minimum-quadratic unbiased estimator of variance components for the repeated measurements model by a new approach

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Keywords:
Minimum norm quadratic unbiased estimator (MINQUE)
Minimum variance quadratic unbiased estimator (MIVQUE)
Repeated measurement model (RMM)
Variance Components

ABSTRACT

Quadratic Unbiased Estimator (MIVQUE), and Minimum Norm Quadratic Unbiased Estimator (MINQUE) satisfying some optimum properties and give some examples for repeated measurements model. Also simulation approach is used.

1. Introduction

The estimation of variance components is an important inference problem in repeated measurements models. There are different methods have been proposed to estimate the variances. Such as MIVQUE and MINQUE which proposed by Rao (1971a ,1971b and 1972). Rao (2002) and Subramani (2012) are introduced a modified for the two methods of variance components in mixed linear models. Further, the concept of RMM has evolved considerably to use in wide scientific disciplines, in particular, the health, agricultural and industrial areas. These models could be applied in animal eugenics studies and even in the improvement of machines performance in the industrial sector.

The model of Vonesh and Chinchilli (1997) that has been modified is an important and widely used in RMM. But the absence of knowledge for values of parameters of the variance components, play a key role for more of researches. Therefore, we proposed a new approach to calculate matrices of MIVQUE and MINQUE to find a suitable estimator for these unknown parameters of the variance components. It is important to note that the two methods MIVQUE and MINQUE are most widely used to estimate the components of variance. Apart from the first and second order moments of the observables, this method does not require any distributional assumptions.

3. The Repeated Measurements Model (RMM): [3], [20]

Consider the linear model of repeated measurement with one between–units factor incorporating univariate random effects, and parameterization. Suppose that repeated measurements at p time points are obtained from q groups of subjects. Let n_j denote the number of subjects in group j, and let Y_{ijk} denote the response at time k from the ith subject in group j.

2.1. Setting up the model:

Consider the following model

$$Y_{ijk} = \mu + \tau_j + \delta_{i(j)} + \gamma_k + (\tau\gamma)_{jk} + \varepsilon_{ijk}, \quad (1)$$

where
$i = 1, \dots, n_j$ is an index for experimental unit within group j,
$j = 1, \dots, q$ is an index for levels of the between-units factor (group),
$k = 1, \dots, p$ is an index for levels of the within-units factor (time),
Y_{ijk} is the response measurements at time k for unit i within group
μ is the overall mean,
τ_j is the added fixed effect for treatment group j,
$\delta_{i(j)}$ is the random effect due to experimental unit i within treatment group j,
γ_k is the added fixed effect for time k,
$(\tau\gamma)_{jk}$ is the added fixed effect for the group $j \times$ time k interaction, and
ε_{ijk} is the random error on time k for unit i within group j.

For the parameterization to be of full rank, we impose the following set of conditions:

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Research Article

The Analysis of Bifurcation Solutions by Angular Singularities

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Abstract. This paper studies a nonlinear wave equation's bifurcation solutions of elastic beams situated on elastic bases with small perturbation by using the local method of Lyapunov-Schmidt. We have found the Key function corresponding to the functional related to this equation. The bifurcation analysis of this function has been investigated by the angular singularities. We have found the parametric equation of the bifurcation set (caustic) with the geometric description of this caustic. Also, the critical points' bifurcation spreading has been found.

Keywords. Bifurcation solutions; Angular Singularities; Caustic

MSC. 34C23; 34F10; 37G10

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1. Introduction

The nonlinear problems which occur in mathematics and physics may be formed in the form of operator equation

$$f(x,\lambda)=b, \quad x\in O,\ b\in Y,\ \lambda\in\mathbb{R}^n. \tag{1}$$

In which f is a smooth Fredholm map whose index is zero and X, Y are *Banach's* spaces and $O\subseteq X$ is open. The method of reduction for these problems to the finite dimensional equation

$$\Theta(\xi,\lambda)=\beta, \quad \xi\in M,\ \beta\in N \tag{2}$$

may be used, where M and N are smooth finite dimensional manifolds. Lyapunov-Schmidt method can reduce equation (1) to equation (2), in which equation (2) has all the analytical and

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Research Article

The Analysis of Bifurcation Solutions for the Camassa-Holm Equation

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Abstract. This paper studies the Camassa-Holm equation's bifurcation solutions by using the local method of Lyapunov-Schmidt. The Camassa-Holm equation has been studied with the formula ODE. We have found the key function corresponding to the functional related to this equation. The bifurcation analysis of this function has been investigated by the boundary singularities. We have found the parametric equation of the bifurcation set (caustic) with the geometric description of this caustic. Also, the critical points' bifurcation spreading has been found.

Keywords. Camassa-Holm equation; Bifurcation solutions; Boundary Singularities; Caustic

MSC. 34K18; 34K10

Received: June 25, 2019 **Accepted:** July 18, 2019

1. Introduction

The nonlinear problems which occur in mathematics and physics may be formed in the form of operator equation,

$$f(x,\lambda) = b, \quad x \in O,\ b \in Y,\ \lambda \in \mathbb{R}^n \tag{1}$$

in which f is a smooth Fredholm map whose index is zero and X, Y are *Banach's* spaces and $O \subseteq X$ is open. The method of reduction for these problems to the finite dimensional equation,

$$\Theta(\xi,\lambda) = \beta, \quad \xi \in M,\ \beta \in N \tag{2}$$

may be used, where M and N are smooth finite dimensional manifolds. Lyapunov-Schmidt method can reduce equation (1) to equation (2) in which equation (2) has all the analytical and

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Weak Galerkin finite element method for solving one-dimensional coupled Burgers' equations

Ahmed J. Hussein & Hashim A. Kashkool

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Abstract

In this paper, we apply a weak Galerkin method for solving one dimensional coupled Burgers' equations. Based on a conservation form for nonlinear term and some of the technical derivational. Theorticly, we drive the optimal order error in L^2 and H^1 norm for both continuous and discrete time weak Galerkin finite element schemes, also the stability of continuous time weak Galerkin finite element method is proved. Numerically, the accuracy and effectiveness of the weak Galerkin finite element method are illustrated by using Numerical examples with the lower order Raviart–Thomas element RT_k for discrete weak derivative space.

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Articles

A weak Galerkin finite element method for two-dimensional coupled burgers' equation by using polynomials of order $(k, k-1, k-1)$

Ahmed Jabbar Hussein & Hashim A. Kashkool

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Abstract

In this paper, we suggest a Weak Galerkin Finite Element Method (WGFEM) for solving two dimensional coupled Burgers' equations with a stabilization term and spicail trilinear form for nonlinear term . We introduce two numerical scheme, the first continuous time weak Galerkin finite element method (CTWGFEM) and the second discrete time weak Galerkin finite element method (DTWGFEM), As well, we use a polynomial mixtue that reduces the number of unknowns in the numerical scheme with no change in the preciseness of the numerical approximation [1]. For clarification purpose, We use polynomial space of degree on each element, polynomial space of degree on the edge and polynomial space of degree for space of weak gradient operator of each element. Convergence order estimate in trip-barnorm and norm it was received. Theoretical proofs matched with numerical results by test problem.

Subject Classification: Primary 65N15 Secondary 65N30

Keywords: Weak Galerkin Finite Element Method (WGFEM) Two-dimensional coupled Burgers' Equations Optimal order

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Articles

The error analysis of linearized discontinuous Galerkin finite element method for incompressible miscible displacement in porous media

Jawad Jaaywel Saadoon & Hashim Abdul-Khaliq Kashkool

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Abstract

We present a linearization scheme for discontinuous Galerkin method for solving the fully coupled one-phase flow problem. The pressure and concentration equations are solved by symmetric interior penalty Discontinuous Galerkin (SIPG) method and we discretized the model using (DGM) space and backward Euler in time. We prove the error estimate in the energy norm.

Subject Classification: Primary 93A30 Secondary 49K15

Keywords: Discontinuous Galerkin method Incompressible miscible displacement Error estimates

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A New Technique to Solve Two-Dimensional Viscous Fluid Flow Among Slowly Expand or Contract Walls

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Keywords:
Yang transform, homotopy perturbation method, 2D viscous flow, convergence analysis

ABSTRACT

In this research, we have proposed a new technique to solve two-dimensional (2D) viscous fluid flow among slowly expanding or contracting walls. The new technique depends on combining the algorithms of Yang transform and the homotopy perturbation methods. The results, obtained from the first iteration and by using the new method, show the accuracy and efficiency of this method compared to the other methods, used to find the analytical approximate solution for the problem caused by the 2D viscous fluid flow. Moreover, the graphs of the new solutions show the validity, usefulness and necessity of the new method.

1. INTRODUCTION

One of the most common problems of the fluid flow and which has interested many researchers is the laminar flow of viscous fluid through a porous channel or pipe with contracting or expanding permeable walls. This great interest is due to its biological applications, like the conveyance of biological fluids through expanding or contracting receptacles, nomination in the lungs and kidneys, flow inside the lymphatic's, and many others besides. Many scientists and researchers have attempted to find solutions for these equations using different methods; for example, the first scientists who found a solution to the two-dimensional laminar flow of a viscous fluid problem in a parallel-walled channel is Berman [1]. He found the solution to this problem by using the perturbation method. He utilized the Reynolds number as a perturbation parameter, where the solution is valid for small values of the Reynolds number. Ganji et al. [2] applied the homotopy perturbation method (HPM) to solve 2D viscous fluid flow problem between slowly expanding or contracting walls. Comparing their results with the numerical method (NM), they noted that HPM is efficacious and simple and can be used to solve nonlinear problems. Moreover, their results demonstrate that HPM is capable to solve this problem with quickly convergence approximations without any restrictive supposition or transformations, which lead to changes in the physical definition of the problem. Dinarvand [3] found a solution for the viscous fluid flow through slowly expanding or contracting walls by using the differential transform method. He compared between his results and the ones obtained from the numerical method to find that his results exhibit the marked accuracy. Moreover, from these results, he noticed that the differential transform method does not demand small parameters in the equations, therefor, this method can be applied to many nonlinear differential and integral equations without perturbation, linearization, or discretization. Dinarvand et al. [4] used the homotopy analysis method (HAM) to find the solutions for the viscous fluid flow with expanding or contracting gaps. They compared the obtained results with the numerical method and the results demonstrate a remarkable accuracy. This method (HAM) differs from perturbation methods in that it does not rely on small parameters. So, it is valid for both mighty and weak nonlinear problems. Sushila et al. [5] used the Sumudu transform homotopy perturbation method (HPSTM) to solve the problem of the viscous fluid flowing between slowly expanding or contracting walls. Numerical results have shown that these methods are very effective and able to solve this problem. Moreover, they observed that this method was able to find the solution without any restrictive suppositions to the rise of changes in the physical definition of the problem. They also noted that the advantage of this method is that it solves nonlinear problems without utilizing Adomian's polynomials. Ledari et al. [6] solved the viscous fluid flow problem by slowly expanding or contracting walls by using Akbari Ganji's Method (AGM). They compared the results of AGM with those of the Ruge-Kutta method, the Adomian decomposition method, the homotopy perturbation method and variational iteration method; the results show that this method is efficacious and has enough accuracy.

Despite all these advantages of the methods, used by researchers to solve the current problem (as illustrated in available literature), there are some disadvantages to these methods, for example, some of these methods require high iterations to obtain an accurate solution to the current problem. Also, some of these methods require a great deal of time and effort to solve the problem.

There are many analytical methods, some of which need a perturbation parameter and others do not need one. One of these methods that do not need perturbation parameters is the Yang transform method (YT), which is a new integral transform proposed in 2016 by Yang [7]. It was first applied to the heat transfer equation in the steady-state. Note that this method is precise and efficacious in finding the analytical solutions to linear differential equations (partial), and it is used by many researchers to solve different problems [7, 8]. In 2018,

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Simultaneous Approximation of New Sequence of Integral Type Operators with Parameter δ_0

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Abstract. In this paper, we define a new sequence of linear positive operators of integral type $W_n(f;x)$ to approximate functions in the space $C_\alpha[0,\infty), \alpha > 0$. First, we study the basic convergence theorem in simultaneous approximation and then study Voronovskaja-type asymptotic formula. Then, we estimate an error occurs by this approximation in the terms of the modulus of continuity. Next, we give numerical examples to approximate two test functions in the space $C_\alpha[0,\infty)$ by the sequence $W_n(f;x)$. Finally, we compare the results with the classical sequence of Szãsz operators $S_n(f;x)$ on the interval $[a,b]$. It turns out that, the sequence $W_n(f;x)$ gives better results than the results of the sequence $S_n(f;x)$ for the two test functions using in the numerical examples.

2010 Mathematics Subject Classifications: 41A10, 41A25,41A36

Key Words and Phrases: Linear positive operators, Simultaneous approximation, Voronovskaja-type asymptotic formula, Modulus of continuity

1. Introduction

Bernstein in 1912, using a sequence known by his name, Bernstein sequence, which is defined as:[1]

$$B_n(f;x) = \sum_{k=0}^{n} b_{n,k}(x) f\left(\frac{k}{n}\right) \tag{1.1}$$

where, $b_{n,k}(x) = \binom{n}{k} x^k (1-x)^{n-k}$ and $f \in C[0,1]$.

Next, Voronovskaja in 1932 shown that the order of approximation is $O(n^{-1})$. Also, she showed that this order of approximation by Bernstein sequence cannot be improved beyond $O(n^{-1})$.[18]. Many papers interested in the classical sequences of Bernstein and gave some modifications of them [5], [11]. In addition, the numerical application for this

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Simultaneous approximation by a new sequence of integral type

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Iraq

Abstract

In our present work, we define a new sequence of linear and positive operators of integral type to approximate unbounded function in the interval [0, ∞). First, we study the ordinary approximation then, we proceed to study the simultaneous approximation of this sequence. In addition, we drive a Voronovskaja-type asymptotic formula of s-times differentiable functions. We support our study with a numerical example to approximate a first derivative of the test function. Also, we compare the results with some sequences which are studied in the past.

Subject Classification: *41A25, 41A28.*

Keywords: *Linear positive operators, Korovkin theorem, Simultaneous approximation, Voronovaska-ja-type asymptotic formula.*

1. Introduction

In 1885, Weierstrass [10] introduced the well-known fundamental theorem in approximation theory called "Weierstrass approximation theorem", in the proof of this theorem, Weierstrass introduced the following sequence:

$$W_n(f;x)=\frac{1}{I_n}\int_{a_1}^{b_1} f(t)e^{-n(t-x)^2}\,dt,$$

where $a_1 = a - \delta$, $b_1 = b + \delta$, $I_n = \int_{-c}^{c} e^{-nt^2}\,dt$, $c = b_1 - a_1$ and $f \in C[a, b]$.

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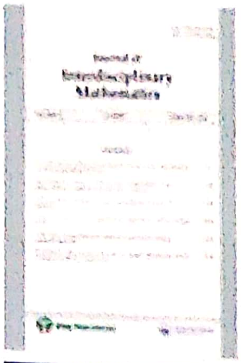

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Improving the order of approximation by a generalization of new bivariate sequence of integral type operators with parameters

Ali J. Mohammad & Hadeel O. Muslim

On spectral asymptotic for the second-derivative operators

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Abstract. In this work we focus on spectral asymptotic for the second derivative operators. Here we study Schrödinger operator with zero-range potentials, because this operator has great importance for understanding the solvable problems in quantum mechanics and atomic physics. It appears in different models such as the mathematical physics, applied mathematics and theoretical physics. We have two objectives in this work. We first demonstrated that this operator has a continuous spectrum contains an infinite number of bands separated by gaps. We then explained that the bands to gaps ratio tends to zero under certain conditions.

1. Introduction

The differential operators are ubiquitous in many natural systems, ranging from quantum to atomic physics applications. These applications are used to give rise a solvable model of complicated physical phenomena [1,2,5]. Because the method of solid-state physics reproduces the geometry of the problem extremely well, therefore, there is a particular interest in the applications of these models. Kroing and Penney [10] were the first who described this model by the Hamiltonian operator

$$\mathrm{H} = -\frac{d^2}{dx^2} + \sum_{n\in Z} \alpha_n \,\delta(x-n),$$

where δ is the Dirac delta function and α_n are the actual coupling constants that describes each point interactions. They also explained the spectrum of permissible energy values which consists of continuous region separated by finite intervals. Further, this operator is used to solve the complicated physical phenomena. The point interactions found in many different models by considering boundary conditions at the individual points. The generalized point interaction in one dimension with boundary conditions

$$\begin{pmatrix}\psi(0^+)\\ \psi(0^+)\end{pmatrix} = e^{i\theta}\begin{pmatrix}\alpha & \beta\\ \gamma & \delta\end{pmatrix}\begin{pmatrix}\psi(0^-)\\ \psi(0^-)\end{pmatrix},$$

is studied in [12, 13]. He also discussed the existence and the physical properties of the one-dimensional δ′-interaction Hamiltonian. Bloch theorem is used to explain that any such operator coincides with some self-adjoint extension of the unperturbed second-derivative operator restricted to the set of functions vanishing in a neighbourhood of the origin [7]. Moreover, the

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Essential self-adjointness of the Schrödinger operator with electromagnetic potential

Yahea Hashem Saleem & Hadeel Ali Shubber

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The Smoothness of Schrödinger Operator With Electromagnetic Potential

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Abstract. In this paper, we prove that the Feynman-Kac Itô formula of the Schrödinger operator with electromagnetic $\Psi(t,x)$ in equation (1) in [8] which defined as

$$\Psi(t,x) = \int d\mu_x^t(\omega)\exp\left(-i\int_0^t b(\omega(s))d\omega - \frac{i}{2}\int_0^t divb\omega(s)ds - \int_0^t V(\omega(s)ds\right)\varphi(\omega(t))$$

is differentiable of the variable t, and so establish that the infinitely differentiable in a region, therefore, investigate smoothness of this function.

2010 Mathematics Subject Classifications: 35J10, 35B65

Key Words and Phrases: Schrödinger operator, electric potential, magnetic potential, smoothness.

1. Introduction

The problem of the self-adjoint operator is central in the quantum machine (the Diracvon Neumann formulation of quantum mechanics, in which physical observables such as position, momentum, angular momentum).

Kato [5] who showed on the basis of his elegant inequality that, if $V(x) \geq 0$ and $V \in L^2_{loc}$, then the Schrödinger operator is essentially self-adjoint on the set of infinitely differentiable finite functions. Nextly, Gaysinsky, Goldstein [4] they proved smoothness of the Schrödinger operator which is one important step to prove self-adjointness must be smoothness. After that, Adam Ward [1] investigated the essential self-adjointness of Schrödinger operator.

Many researchers studied self-adjoint operator were done, for example [2], [6], [7], [9].

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Analysis of error estimate for expanded H^1 - Galerkin MFEM of PIDEs with nonlinear memory

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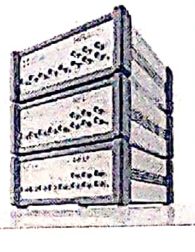
Hameeda Oda Al-Humedi, and Ali Kamil Al-Abadi



AIP Conference Proceedings 2235, 020010 (2020); https://doi.org/10.1063/5.0007637 2235, 020010

AIP Conference Proceedings

The numerical solution of time-space fractional bioheat equation by using fractional quadratic spline methods

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Ammar Muslim Abdulhussein, and Hameeda Oda

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AIP Conference Proceedings

Spectral shifted Jacobi-Gauss-Lobatto methodology for solving two-dimensional time-space fractional bioheat model

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Hameeda Oda Al-Humedi, and Firas Amer Al-Saadawi

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A New Mixed Nonpolynomial Spline Method for the Numerical Solutions of Time Fractional Bioheat Equation

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Abstract

In this paper, a numerical approximation for a time fractional one-dimensional bioheat equation (transfer paradigm) of temperature distribution in tissues is introduced. It deals with the Caputo fractional derivative with α order for time fractional derivative and new mixed nonpolynomial spline for second order of space derivative. We also analyzed the convergence and stability by employing Von Neumann method for the present scheme.

Keywords: Fractional bioheat equations, Caputo fractional derivative, new mixed nonpolynomial spline, stability.

الطريقة الجديدة Mixed Nonpolynomial Spline لإيجاد الحلول العددية لمسألة الانتشار الحراري الحيوي الكسورية بالنسبة للزمن

عمار مسلم عبد الحسين[1]* ، حميدة عودة الحميدي[2]
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الخلاصه

في هذه المقالة، تم حل عدديا لمسألة الأنتشار الحراري الحيوي الكسورية الزمانية لتوزيع درجة الحرارة في الأنسجة باستخدام الطريقة الجديدة Mixed Nonpolynomial Spline وتم استخدام طريقة الأشتقاق الكسوري Caputo من الرتبة α للمشتقة الزمانية والطريقة الجديدة بالنسبة للمشتقة المكانية ثم تمت مقارنة بين الحلول الدقيقة والعددية. ثم ناقشنا تحليل الاستقرارية بطريقة فون نيومان.

1. Introduction

In the human body, the skin is considered as the largest organ. The study of skin and thermal behavior of living tissues is very fundamental, and it can be mathematically described by Pennes' bioheat transport equation:

$$\rho_t c_t \frac{\partial T(x,t)}{\partial t} = \mu \frac{\partial^2 T(x,t)}{\partial x^2} + W_b c_b (T_a - T) + Q + q_m \tag{1}$$

Mathematical resolve of the complex thermal interaction between the vasculature and tissues is a topic of interest for numerous physiologists, physicians, and engineers [1]. Temperature distribution in skin tissues is important for medical applications such as skin cancer, skin burns, etc. [2]. At most, the

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The Numerical Approximation of the Bioheat Equation of Space-Fractional Type Using Shifted Fractional Legendre Polynomials

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Received: 17/8/ 2019 Accepted: 30/9/2019

Abstract
The aim of this paper is to employ the fractional shifted Legendre polynomials (FSLPs) in the matrix form to approximate the fractional derivatives and find the numerical solutions of the one-dimensional space-fractional bioheat equation (S-FBHE). The Caputo formula was utilized to approximate the fractional derivative. The proposed methodology applied for two examples showed its usefulness and efficiency. The numerical results showed that the utilized technique is very efficacious with high accuracy and good convergence.

Keywords: Collocation method, Space-Fractional bioheat equation, Fractional shifted Legendre polynomials, Numerical accuracy.

متعددات حدود ليجندر الكسورية التقريب العددي لحل معادلة الكسوريه بأستخدام *Bioheat*

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الخلاصة
الهدف من هذا البحث هو توظيف متعددات الحدود ليجندر الكسورية (FSLPs) بالشكل المصفوفي لتقريب المشتقات الكسورية لإيجاد الحلول العددية لمعادلة bioheat الكسورية أحادية البعد (S-FBHE) . أستخدمنا صيغة Caputo لتقريب المشتقة الكسورية . تبين المنهجية المقترحة المطبقة على مثالين فائدتها وكفائتها. تظهر النتائج العددية أن التقنية المستخدمة فعالة للغاية، وتعطي دقة عالية وتقاربا جيدًا.

1. Introduction

Many problems in various fields can be successfully modeled by the ordinary, partial or fractional differential equations. Fields of application include, for example, biomedical engineering, physics, viscoelasticity, biology, and fluid mechanics ,etc. In many cases, finding the exact solutions for these equations is difficult or impossible. Therefore, researchers used approximate or numerical solution methods [1-5].
The fractional calculus is utilized to improve the modeling accuracy of many phenomena in natural sciences. The most important merit of utilizing fractional differential equations is their non-local

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Bayesian Panel Data Model with Constraints

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Abstract. In this paper, we consider a set of linear constraints on the coefficients of the random panel data model. Furthermore, Bayesian approach based on Markov Chain Monte Carlo (MCMC) is employed to making inferences on the model coefficients subject to this constraints.

Keywords. Panel Data Model , Maximum Likelihood Method ,constrained maximum likelihood estimator, Prior distribution , Posterior distribution, Bayesian estimation.

1. Introduction

The analysis of panel data allows the model builder to learn about economic processes while accounting for both heterogeneity across individuals, firms, countries, and so on and for dynamic effects that are not visible in cross sections. Modelling in this context often calls for complex stochastic specifications,[7]. The panel data model has been investigated by the many researcher as Elhorst in (2001) presented paper surveys panel data models extended to spatial error autocorrelation or spatially lagged dependent variable, [5]. Hurlin in (2004) proposed a simple test of Granger (1969) non causality hypothesis in heterogeneous panel data models with fixed coefficients,[10]. Bun , a. e. in (2005) studied extends earlier results on bias – corrected estimators for the fixed effects dynamic panel data model,[3]. Gorgens a. e. in (2008)discussed efficient estimation of nonlinear dynamic panel data models with application to smooth transition models ,they explores estimation of a class of nonlinear dynamic panel data models with additive unobserved individual specific effects,[9]. Feng a. e. in (2015) proposed a panel data Semiparametric varying coefficient model in which covariates (variables affecting the coefficients) are purely categorical,[6]. Ashley and Sun in (2016) proposed subset continuous updating GMM estimators for dynamic panel data models,[1].

The cornerstone of Bayesian methodology is the Bayes theorem, which is known as the principle of inverse probability. It helps us make probability statements about parameters after the sample has been taken. The conditional distribution of the parameters after observing the data is the posterior distribution that summarizes the prior and the sample information posterior information is proportional to sample information times prior knowledge. Attainment of the posterior is only the beginning of the research methodology since the statistical inference will be based on the posterior and predictive distributions that are derived using the Bayes' rule. However, to obtain the posterior we do need the data and the prior distribution. The choice of the prior distribution depends on the knowledge of the investigator as well as his willingness to incorporate beliefs and theoretical postulates into the methodology. There are explicit rules for selecting prior distributions whether an informative or non-informative prior is preferred. In the spirit of the simplicity postulate, it is reasonable to begin with a simple case, a regression model with a constant term and a regressor. Markov Chain Monte Carlo (MCMC) is both feasible and provides sufficiently accurate results if used with care. Based on the problem at hand, the investigator can utilize either of these tools or a combination of them can be constructed. This makes it possible to sample from the complicated posterior distributions and/or compute posterior moments or any other inferential summary statistic. Calculation of the marginal posterior functions is an important part of Bayesian analysis, for the usual objective is to make inferences about individual parameters and or provide graphs for those marginal posterior densities. MCMC methods facilitate such investigations,[4],[12],[13],[14].

The linear model subject to a set of linear constraints on the coefficients of the model arises commonly in applied econometrics as well as other scientific applications. Constrained parameter

First Integral Method for Constructing New Exact Solutions of The important Nonlinear Evolution Equations in Physics

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Abstract
In this paper, some new exact solutions of the important nonlinear partial differential equations in physics as Gardener's equation and Sharma-Tasso-Over equation are formally derived by utilizing the first integral method, where it is equipment us with many exact solutions by using the travelling wave transform, then deduce a system of ordinary differential equations which is solved by depending on theorem in commutative algebra and with helping the mathematical software like Maple and Wolfram Mathematica.

Keywords
First integral method; Gardener's equation and Sharma-Tasso-Over equation, a theory of commutative algebra.

1. Introduction
In the recent years, A matter of getting the exact solution of nonlinear partial differential equations (NLPDEs) has aroused the interest of many scientists, due to the appearance of these equations in many scientific fields such as engineering complex physics phenomena, mechanics, chemistry and biology etc, also as a result of the development in the field of computer software like Maple or Mathematica, which enables us to perform the complicated and tedious algebraic calculations easily and high efficiency, moreover, by the exact solutions, we can easily verify the accuracy and validity of the numerical solutions and also analyses the stability of these solutions, so many efficient analytical methods have emerged to find the exact solutions of nonlinear evolution equations had proposed such as tanh method was applied by Khater et al. [9], tanh-sech method Malfliet [11], extended tanh method by El-Wakil and Abdou [3], sine-cosine method Wazwaz [18, 19], F-expansion method Sheng [15], the extended mapping method Peng and Krishnan [20], the exp(-G(ξ)) Method Fengyan [7] etc.

Feng [6], proposed a new powerful method, which called the first integral method for solving Burgers-KdV equation. This method depends on the concept of the theory of commutative algebra Ding and Li [2]. The magnificence of this method embodies in the enjoyment of the following advantages, firstly, it avoids a great deal of tiresome and complicated calculations and the second point it supplies different more exact and explicit travelling solitary solutions, moreover, it has proved ease and applicability for different types of differential equations, so it is considered an easier and quicker method than other traditional techniques. Recently this useful method was applied to solve fractional equation Eslamil et al. [5] also used widely by many researchers [1, 10, 12, 13, 14, 16, 17]

In the present work, we would like to extend the application of the first integral method to solve important equations are Gardener's equation and Sharma-Tasso-Over equation. The Structure of this article can be arranged as follows: Section 2, gives a short introduction to the first integral method. Applying the first integral method and some new exact solutions are obtained for nonlinear partial differential equation (PDE) as Gardner's equation and the Sharma- Tasso- Olver equation In section 3. Finally, in section 4, the conclusion of this research is summarized.

2. The basic idea of the first integral method(FIM)

We recap the main points of FIM by considering a general nonlinear PDE in the form

$$P(u, u_t, u_x, u_y, u_{tt}, u_{xt}, u_{yt}, u_{xy}, u_{xxx}, \dots) = 0, \tag{2.1}$$

Topological Data Analysis for Image Forgery Detection

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Abstract

The manipulation of digital images has become easy due to powerful computers, advanced photo-editing software packages and high-resolution image-capturing devices. The identification of image authenticity has received much attention because of the increasing power of image editing methods. Object removal is an image forgery technique, which is usually achieved by the Exemplar-Based Inpainting (EBI) method without any noticeable traces. Some legal issues may arise when a tampered image cannot be distinguished from a real one by visual examination. Therefore the manipulation of digital images has become a huge challenge to passive image forensics. There are a lot of forgery techniques that use to detect on these images after removing the object, but these techniques have limitations when used some post-processing operations such as super-resolution processing, noise addition, blurring and compression processes. To address this issue, this paper proposes a novel forgery detection technique to recognize tampered inpainting images, using topological data analysis (TDA) approach. TDA is a mathematical approach concern studying shapes or objects to gain information about connectivity and closeness property of those objects. This proposed technique is applied for a large number of natural images with getting a good results.

Keywords: *Image forgery. Image inpainting. Topological Data Analysis. Local Binary patterns. kNN classifier.*

Introduction

In recent years, the field of digital image forgery detection has remained active and has received significant interest from the scientific community. A wide-ranging study about image forgery detection have been introduced in [(1)].

A copy-move and object removal are a famous forgeries processes at this time. There are a lot of techniques for detecting the copy-move regions in images in the literature, also there are a lot of techniques for detecting the forgery regions in images. However, we will focus on techniques for the detection of object removals in an image, which is usually completed by EBI method in an unnoticeable way. Up to now, there are few works which report about the blind detection of image inpainting [(2), (3), (4), (5), (6)].

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As the first attempt, the authors in [(2)] introduced a forgery detection method for EBI based on zero-connectivity features, and fuzzy membership is proposed to detect specific image doctoring to yield the degree of matching of blocks in suspicious regions and identified forged regions by the fuzzy membership. However, this method failed to detect the forged regions in the compressed and JPEG images.

Later, the authors in [(3)] presented an automatic forgery detection method for EBI process. The proposed method contains two major processes: suspicious region detection and forged region identification. The proposed method performs well with regard to both accuracy and speed while detecting forged regions. However, this method still incapable to locate the forged regions of small sizes.

Therefore, Bacchuwar et al. in [(4)] proposed a novel method to detect inpainting forgery and copy-move regions using a jump patch-block matching, which makes this method robust and faster than the already existing methods. This method reduces computational costs.

PERIÓDICO TCHÊ QUÍMICA
ARTIGO ORIGINAL

UM NOVO MÉTODO ANALÍTICO PARA RESOLVER A EQUAÇÃO DE TELEGRAFIA LINEAR E NÃO-LINEAR

A NOVEL ANALYTIC METHOD FOR SOLVING LINEAR AND NON-LINEAR TELEGRAPH EQUATION

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RESUMO

A modelagem de muitos fenômenos em vários campos, como matemática, física, química, engenharia, biologia e astronomia, é feita pelas equações diferenciais parciais não lineares (PDE). A equação do telégrafo hiperbólico é uma delas, onde descreve as vibrações de estruturas (por exemplo, edifícios, vigas e máquinas) e é a base para equações fundamentais da física atômica. Existem vários métodos analíticos e numéricos para resolver a equação do telégrafo. Uma solução analítica considera enquadrar o problema de uma forma bem compreendida e calcular a resolução exata. Também ajuda a entender as respostas para o problema em termos de precisão e convergência. Esses métodos analíticos têm limitações com precisão e convergência. Portanto, um novo método analítico aproximado é proposto para lidar com restrições neste artigo. Este método usa as séries de Taylors em sua derivação. O método proposto foi usado para resolver a equação hiperbólica de segunda ordem (equação Telegraph) com a condição inicial. Três exemplos foram apresentados para verificar a eficácia, precisão e convergência do método. As soluções do método proposto também foram comparadas com as obtidas pelo método de decomposição adomiana (ADM) e pelo método de análise de homotopia (HAM). A técnica é fácil de implementar e produz resultados precisos. Em particular, esses resultados mostram que o método proposto é eficiente e melhor que os outros métodos em termos de precisão e convergência

Palavras-chave: *Equação do Telégrafo Hiperbólico, Série de Taylor, Solução Analítica, Precisão, Operador Não Linear.*

ABSTRACT

The modeling of many phenomena in various fields such as mathematics, physics, chemistry, engineering, biology, and astronomy is done by the nonlinear partial differential equations (PDE). The hyperbolic telegraph equation is one of them, where it describes the vibrations of structures (e.g., buildings, beams, and machines) and are the basis for fundamental equations of atomic physics. There are several analytical and numerical methods are used to solve the telegraph equation. An analytical solution considers framing the problem in a well-understood form and calculating the exact resolution. It also helps to understand the answers to the problem in terms of accuracy and convergence. These analytic methods have limitations with accuracy and convergence. Therefore, a novel analytic approximate method is proposed to deal with constraints in this paper. This method uses the Taylors' series in its derivation. The proposed method has used for solving the second-order, hyperbolic equation (Telegraph equation) with the initial condition. Three examples have presented to check the effectiveness, accuracy, and convergence of the method. The solutions of the proposed method also compared with those obtained by the Adomian decomposition method (ADM), and the Homotopy analysis method (HAM). The technique is easy to implement and produces accurate results. In particular, these results display that the proposed method is efficient and better than the other methods in terms of accuracy and convergence.

Keywords: *Hyperbolic Telegraph Equation, Taylors' Series, Analytical Solution, Accuracy, Non-Linear Operator.*

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A Unique Fixed Point Using Fuzzy Cone and ξ-Fuzzy Cone Integrable Functions

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Abstract: The purpose of this study is to present the idea of an f-partition for a closed interval. Also, we introduce a new notion to a notion of fuzzy cone metric space that is called fuzzy cone function as well we prove fundamental fixed-point theorems. Moreover, we obtain a concept of ξ-fuzzy cone integrable function. Finally, we conclude new results for fixed point in the ξ-fuzzy cone integrable functions.

Key words: Fuzzy cone metric space, f-partition for (u, v), fuzzy cone integrable function, ξ-h function, ξ-fuzzy cone integrable function and fixed-point theorems, fundamental

INTRODUCTION

The idea of the cone metric space and the fixed point in the cone metric space was presented by Huang and Zhang (2007). They replaced the set of the real numbers by an ordered Banach space in the definition of the metric and it is a generalized to the notion of the metric space. Further, there are many researchers discussed the generalization of the fixed point in the cone of these spaces (Abbas and Jungck, 2008; Huang and Zhang, 2007; Vetro, 2007). Di Bari and Vetro (2008) introduced a new concept of the fixed point in the field of φ-mapping and concluded some of the results and examples achieved.

One of their interested on the applications in the fuzzy cone metric space was introduced by Bag (2013) where he studied many subjects that are related to it. After that many researchers touched on the concept of the fixed point in this space. Oner *et al.* (2015) provided another definition of the concept of fuzzy cone metric space and concluded some of the results related to it and fuzzy metric space is studied by many researchers (AL-Mayahi and Hadi, 2015).

In this study, the notion of fuzzy cone metric space is investigated and some fundamental definitions are given. Also, we define a new notion of integration to the notion of fuzzy cone metric space that is called fuzzy cone integrable function. We prove fundamental fixed-point theorems. Moreover, we introduce a concept of ξ-fuzzy cone integrable function. Finally, we conclude new results for the fixed point in the ξ-fuzzy cone integrable function. The range of fuzzy cone metric is considered as such where such is defined by the set of all non-negative fuzzy real number that are defined on $E^*(I)$ where $E^*(I)$ is a given real Banach space (Bag, 2013).

MATERIALS AND METHODS

Background materials: The definitions of the fuzzy number, the convex fuzzy real number and the normal fuzzy number are mentioned by Mizumoto and Tanaka (1979) as well as for the arithmetic operations ⊕, ⊖, ⊙ on E×E were presented by Bag (2015). There are also important definitions such as fuzzy real Banach space, interior point and fuzzy closed subset of such are mentioned by Bag (2013). A subset P of such is called fuzzy cone if:

- P is fuzzy closed, nonempty and $P \neq \{0\}$
- $a, b \in R$, $a, b \geq 0$, $v, w \in P \Rightarrow av \oplus bw \in P$
- $v \in P$ and $\ominus v \in P \Rightarrow v = 0$

Given a fuzzy cone $P \subset E^*(I)$ such, we define a partial ordering ≤ with respect to P by $v \leq w$ if $w \ominus v \in P$. On the other hand $v < w$ such that $v \neq w$ while $v \leq w$ will stand $\ominus v \in \text{Int}\, P$.

A fuzzy cone P is said to be normal if there is a number $k > 0$ such that for all x, $y \in E^*(I)$ such with $0 \leq v \leq w \Rightarrow v \leq kw$.

A fuzzy cone P is said to be regular if $\{x_n\}$ is an increasing sequence which is bounded above is convergent that is $x_1 \leq x_2 \leq, ..., x_n \leq w$ for some $w \in E^*(I)$ such, then there is $x \in E^*(I)$ such that $\|x_n - x\| \to 0$ as $n \to \infty$.

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Fixed Point Theorems of Fuzzy $\mathcal{T}^*$-Cone Metric Space and Their Integral Type Application

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Abstract
The objective of this work is to study the concept of a fuzzy $\mathcal{T}^*$-cone metric space And some related definitions in space. Also, we discuss some new results of fixed point theorems. Finally, we apply the theory of fixed point achieved in the research on an integral type.

Keyword: Fuzzy $\mathcal{T}^*$-cone metric space, Fuzzy-complete, Fuzzy-continuous, Fuzzy-sequentially convergent and fixed point with application of integral type.

مبرهنات النقطة الصامدة في الفضاء المتري الضبابي المخروطي من النوع $\mathcal{T}^*$ مع تطبيقاتها التكاملية

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الخلاصة
الهدف من هذا العمل هو دراسة مفهوم الفضاء المتري الضبابي المخروطي من النوع $\mathcal{T}^*$ وبعض التعاريف المتعلقة بهذا الفضاء. أيضا، سنثبت بعض النتائج الجديدة لنظريات نقطة ثابتة. وأخيرا، نطبق نظرية النقطة الثابتة التي تم تحقيقها حول دالة تكاملية.

1. Introduction

The Cone metric space and the fixed point this space was presented by L. G. Huang and X. Zhang [1]. The Cone metric space is a generalization of metric space. Further, there are many researchers discussed the generalization of the fixed point theorem in Cone of these space [2-7]. In 2008, Bari and Vetro introduced a new concept on the fixed point in the field of φ- mapping and concluded some of the results and examples achieved.

One of their interested in the applications in the fuzzy Cone metric space was introduced by Bag [8] and studied many subjects that are related to it. After that, many authors touched on the concept of the fixed point in this space. Oner, Kandemir, and Tanay provided another definition of the concept of fuzzy cone metric space and concluded many of the results related to it (see[4-6], [10] and[11]).

In this work, we will define fuzzy $\mathcal{T}^*$-Cone metric space. We also mentioned some definitions related to this space and proved some of the properties achieved. In addition, we show fa ixed point in fuzzy $\mathcal{T}^*$-Cone metric space. The range of fuzzy Cone metric is considered as $C^*(I)$, where $C^*(I)$ is defined by the set of all fuzzy real number defined on C where C is a given real Banach space),[9].

2. MAIN RESULTS

The definitions of the fuzzy number, convex fuzzy real number, and normal fuzzy number as mentioned in ([12]), as well as for the arithmetic operations $\oplus$, $\ominus$, $\odot$ on $C \times C$ were presented by

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